

# Free Vibration Analysis of Delaminated Composite Plates

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**ABSTRACT:** Delamination is one of the predominant forms of failure in laminated composite due to lack of reinforcement in the thickness direction. The delamination is common in laminated plates as the composite plates are having laminated structures. Delamination leads to development of cracks which reduces the strength of the material and ultimately the material fails to bear the desirable load. Delaminations as a result of impact for a manufacturing defect can cause a significant reduction in the load carrying capacity of a structure. The simulation of the delamination process of composite laminates is quite complex and requires advanced Finite Element modelling techniques. The modelling and analysis is done in a computationally efficient finite element method using first order shear deformation theory. Delaminations with area  $a/3$  and  $a/2$  at four different interfaces are analysed. Delaminated plates with different boundary conditions such as CCCC, SSSS, SCCC, SSCC and SSSC are considered for the analysis.

**KEYWORDS:** Delamination, composite, FSDT, Boundary conditions, Finite element method

## I. INTRODUCTION

The use of plates in various aerospace, marine, automobile structures and also in infrastructure application such as civil engineering is expanding very fast. A plate is a structural element which is characterized by two key properties. Firstly, its geometric configuration is a three-dimensional solid whose thickness is very small when compared with other dimensions. Secondly, the effects of the loads that are expected to be applied on it generate stresses whose resultants will act exclusively normal to its thickness. One of the major concerns in designing composite made structure is their susceptibility to impact, in certain circumstances; impact can cause severe damage in composite structures, resulting in internal and external failure of the structures such as matrix cracking, fibre breakage and delaminations. Delaminations are the cracks inside interior of the laminate. It is also called barely visible impact damage (BVID), which cannot be easily detected by visual inspection. Delamination may also develop due to shocks, impact loading, repeated cyclic stresses, low velocity impact by foreign objects, dropped tools or bird strikes manufacturing defects such as incomplete wetting and entrapped air bubbles between layers. Knowledge about the natural frequencies and mode shapes of plate with delamination will give an idea about the size and location of delamination.

## II. RELATED WORK

Tanaka et al., [1] studied the integral equation approach for free vibration problems of elastic plate structures. In their study, the boundary integral equation method was used to determine the eigen frequency by means of direct search of the zero-determinant value of the system matrix. This new integral equation approach and its solution give an approximate fundamental solution to the static problem. Della and Shu [2] have presented a review of different method of delaminated plates and beam. The study of delaminated composite structures is mainly categorised into two approaches. First one is region wise approach and second one is Layer-wise approach.

Campanelli and Engblom [3] has developed a finite element model for delaminated composite plates, the model is eight noded shear deformable plate element each node has six degree of freedom and the delaminated plate is

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divided in upper and lower sub-laminates. He uses penalty parameter approach to connect the upper and lower sub laminate to the integral laminate element. Multiple delaminations between the layers through layer wise theory are derived by Barbero and Reddy [4]. They applied the jump discontinuity condition at the interface.

Ju et al., [5] introduced a dynamic finite element technique for free vibration analysis of typically laminated composite beams on the idea of 1st order shear deformation theory. The influences of Poisson impact, couplings among extensional, bending and torsional deformations, shear deformation and rotary inertia are incorporated within the formulation. The dynamic stiffness matrix is formulated primarily based on the precise solutions of the differential equations of motion governing the free vibration of generally laminated composite beam. Jaehong and Lee [6] have also derived the free vibration analysis of delaminated plates. They used the Hamilton principle to derive the equation of motion and finite element analysis for formulate the problem. Based on the classical plate theory Nagesh babu and Hanagud [7] have developed a finite element model for the analysis of free vibration analysis of delaminated smart composite structures. Equation of motion has derived by the help of principle of minimum potential energy and the continuity of the axial displacement slope, moment and shear are satisfied at the delaminated interface.

### III. MATHEMATICAL FORMULATION

The global coordinate system is located at the mid-plane of the plate with the z-axis perpendicular to it. The delaminations are assumed to be parallel to the mid-plane of the plate. Delamination divides the plate into several sub laminates in the delaminated segment.

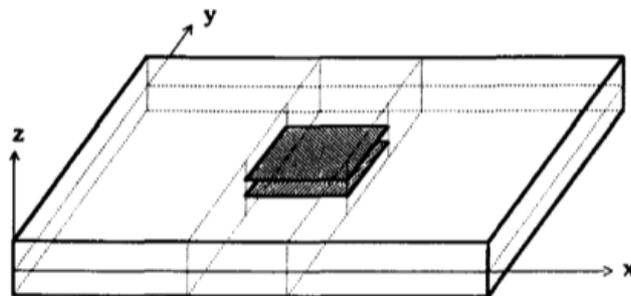


Figure 1: Composite laminate plate with square delamination

An 8-node with 5 degree of freedom element is considered. So an isoparametric plate element with total 40-degree-of-freedom is employed to discretize the delaminated plate with arbitrary 8 layers.

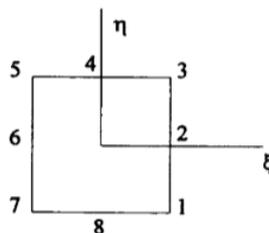


Figure 2: Eight Node serendipity plate element

Displacement model is based on the First order shear deformation theory of laminated composite plates.

$$u(x, y, z) = u_0(x, y) + z\theta_x(x, y)$$

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$$w(x, y, z) = w_0(x, y) + z\theta_y(x, y)$$

$$v(x, y, z) = v_0(x, y) + z\theta_x(x, y)$$

### Delamination model

Laminated plate is divided into sub-laminates. Delaminated and non-delaminated regions are analyzed separately. 'abcd' is delaminated area of laminated plates, so the laminate is also split into upper delamination and lower delamination laminate.

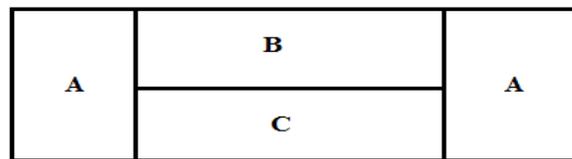


Figure 3: Front view of three segments of delaminated plates

Where A is the integral parts of the laminated plates and B, C are the upper and lower delaminations.

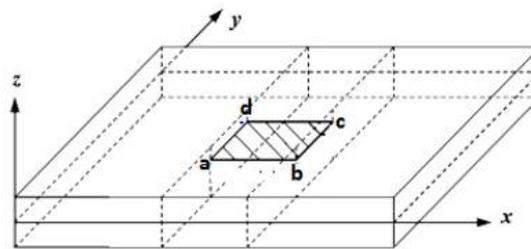


Figure 4: Isometric view of delaminated plate

### Finite element analysis without delamination

Finite element analysis of un-delaminated plate based on the first order displacement theory is as following. An eight noded serendipity plate element and five degree of freedom at each node is presented here for finite element model.

$$u = \sum_{i=1}^8 u_i N_i \quad v = \sum_{i=1}^8 v_i N_i \quad w = \sum_{i=1}^8 w_i N_i \quad \theta_x = \sum_{i=1}^8 \theta_{xi} N_i \quad \theta_y = \sum_{i=1}^8 \theta_{yi} N_i$$

Stress-strain relation in global axis system

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Where N is the in-plane force, M is the in-plane moment,  $\epsilon^0$  is the mid plane strain, K is the curvature.

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$$A_{ij} = \sum_{k=1}^N (Q_{ij})_k (z_k - z_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^N (Q_{ij})_k (z^2_k - z^2_{k-1})$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^N (Q_{ij})_k (z^3_k - z^3_{k-1})$$

$A_{ij}$ ,  $B_{ij}$  and  $D_{ij}$  are laminate extensional stiffnesses, laminate-coupling stiffnesses, and laminate-bending stiffnesses respectively.  $z_k$  is the corresponding distance from middle surface to outer surface of the  $k_{th}$  lamina, and  $z_{k-1}$  is the distance from the middle surface to the inner surface of the  $k_{th}$  lamina.

Lamina Stiffness Matrix,

$$D = \begin{bmatrix} A & B \\ B & D \end{bmatrix}_k$$

By considering the engineering behaviour of the laminate, we combine the element strain matrix, [B] with the ABD matrix and develop into a local stiffness matrix, K which applies for a laminate. The following steps are similar to the conventional FEM, for example the assembly of the element stiffness matrix into global stiffness matrix, before an application of the initial condition and boundary condition for the laminate.

Strain-displacement relation

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 & 0 & 0 \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial y} \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \theta_{xi} \\ \theta_{yi} \end{Bmatrix}$$

Stress-strain relationship for shear forces is

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \sum_{k=1}^N \int_{z_{k-1}}^{z_k} k_1 \begin{bmatrix} Q_{55} & Q_{45} \\ Q_{45} & Q_{44} \end{bmatrix}_k \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}_k dz$$

Here N is number of layer in laminate, k denote the particular layer and  $k_1$  = shear correction factor

Strain-displacement relation is

$$\varepsilon = [B] \{ \delta \}$$

Where

$$[B] = [B_1 \ B_2 \ B_3 \ B_4 \ B_5 \ B_6 \ B_7 \ B_8]$$

$$\{ \delta \} = [\delta_1 \ \delta_2 \ \delta_3 \ \delta_4 \ \delta_5 \ \delta_6 \ \delta_7 \ \delta_8]$$

Interpolation between global coordinate system to natural coordinate system is as follows.

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$$x = \sum_{i=1}^8 N_i(\xi, \eta) x_i \qquad y = \sum_{i=1}^8 N_i(\xi, \eta) y_i$$

Now we have to calculate the element strain energy and kinetic energy and stiffness and mass matrix which are given below.

$$U = \frac{1}{2} \int_{-1}^1 \int_{-1}^1 \{\delta\}^T [B]^T [D] [B] \{\delta\} \det |j| d\xi d\eta$$

$$U = \{\delta\}^T [k] \{\delta\}$$

$$[k] = \int_{-1}^1 \int_{-1}^1 [B]^T [D] [B] \det |j| d\xi d\eta$$

$$T = \frac{1}{2} \int_{-1}^1 \int_{-1}^1 \{\delta\}^T [F]^T [\rho] [F] \{\delta\} \det |j| d\xi d\eta$$

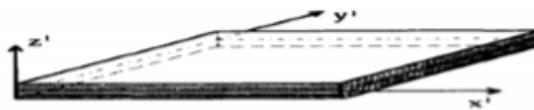
$$T = \{\delta\}^T [m] \{\delta\}$$

$$[m] = \int_{-1}^1 \int_{-1}^1 [F]^T [\rho] [F] \det |j| d\xi d\eta$$

$$F = \begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix} \qquad \rho = \begin{bmatrix} \rho^k & 0 & 0 & 0 & 0 \\ 0 & \rho^k & 0 & 0 & 0 \\ 0 & 0 & \rho^k & 0 & 0 \\ 0 & 0 & 0 & \rho^k & 0 \\ 0 & 0 & 0 & 0 & \rho^k \end{bmatrix}$$

### Finite element analysis of delaminated plates

The upper and lower segment of the delaminated plate is meshes separately which is similar to that used in segment without delamination.



**Figure 5: Coordinate system of delaminated segment of plate**

$x^1, y^1, z^1$  is the local coordinate system for the element. Displacement field is same as first order shear deformation theory, which is taken before, when the segment is without delaminated only the coordinate system is changed. If 'p' is the number of layers in the upper and lower delaminated element and the interface of delamination is middle then the bending stiffness matrix and density matrix is changed like

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<p>Lower delamination bending stiffness</p> $[a_y] = \sum_{k=1}^p [Q_y]_k (Z_k - Z_{k-1})$ $[b_y] = \frac{1}{2} \sum_{k=1}^p [Q_y]_k (Z_k^2 - Z_{k-1}^2)$ $[d_y] = \frac{1}{3} \sum_{k=1}^p [Q_y]_k (Z_k^3 - Z_{k-1}^3)$	<p>Upper delamination bending stiffness</p> $[a_y] = \sum_{k=p+1}^N [Q_y]_k (Z_k - Z_{k-1})$ $[b_y] = \frac{1}{2} \sum_{k=p+1}^N [Q_y]_k (Z_k^2 - Z_{k-1}^2)$ $[d_y] = \frac{1}{3} \sum_{k=p+1}^N [Q_y]_k (Z_k^3 - Z_{k-1}^3)$
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From the above a, b, d matrix, calculate the  $D_1$  matrix of upper and lower delaminated segment element and then the stiffness and mass matrix of the delaminated element is calculated.

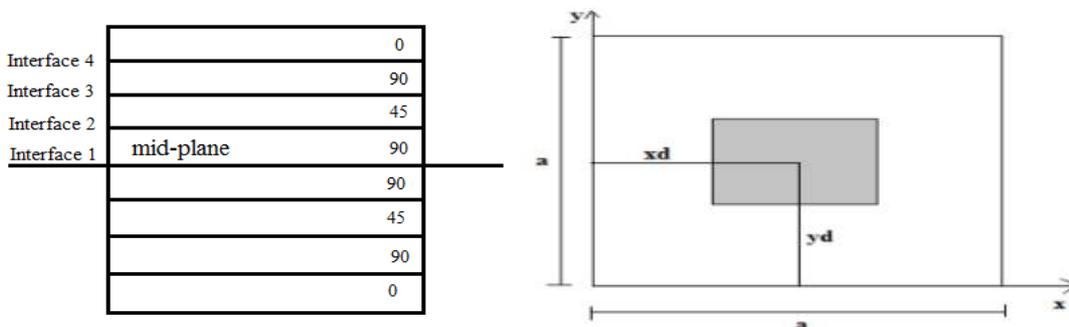
$$[K_1] = \int_{-1}^1 \int_{-1}^1 [B]^T [D_1] [B] \det |J| d\xi d\eta$$

After satisfying the displacement continuity condition at boundary connecting the delaminated and undelaminated segment, we have to assemble the all the element stiffness and element mass matrix and solving the eigen value problem for the free vibration analysis. The eigen value equation is given below.

$$([K] - \omega^2 [M]) \{\lambda\} = \{0\}$$

## IV. VIBRATIONAL ANALYSIS

In this analysis, the effects of delamination size, delamination location, mode of vibration, effect of boundary conditions on the natural frequencies and mode shapes are thoroughly investigated. Numerical results for the eight-layer square composite plate whose material properties, configuration, geometry and boundary condition are taken as  $E_1 = 1.32 \times 10^{11}$  N/m<sup>2</sup>,  $E_2 = 5.35 \times 10^9$  N/m<sup>2</sup>,  $G_{12} = 2.79 \times 10^9$  N/m<sup>2</sup>,  $\rho = 1446.2$  kg/m<sup>3</sup>,  $\nu_{12} = 0.291$ ,  $\nu_{23} = 0.3$ , Orientation of each ply = [0/90/45/90/90/45/90/0]. Plate geometry is  $0.25 \times 0.25 \times 0.00212$  m<sup>3</sup>,  $a=0.25$  m and  $t=0.00212$  m.  $a$ =length of the plate and  $t$ =thickness of the plate. Delamination in plate is assumed at any interface 1, 2, 3, 4 .The delamination location is  $(x_d, y_d)$ ,  $x_d = y_d = a/2$ , interface 1 is treated as mid plane of the plate. Delamination size  $a/2$  and  $a/3$  are considered. The above material properties, geometry and configuration is taken from the Ju et. al. paper[5] for the validation purpose. The boundary conditions considered are CCCC(four edges clamped), SSSS (four edges simply supported), SCCC,SSCC and SSSC.



**Figure 6: Ply delamination position and ply configuration front and top view**

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## V. RESULTS AND ANALYSIS

To investigate the effect of delaminations on the natural frequencies and mode shapes of a delaminated plate, a square delamination is presumed to be located at interface 1, 2, 3 and 4. The size of the delamination is varied and six different kinds of typical boundary conditions are considered. Namely CCCC, SSSS, SSCC, SCCC and SSSC.

**Table I: Natural frequencies (Hz) of CCCC plate with a square delamination of side a/3 at different interfaces**

Condition		Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
Without delamination	present	345.61	641.77	772.31	976.29	1152.3	1397.8
	Ju et al.	346.59	651.51	781.06	1017.2	1195.4	1487.0
Interface 1	Present	341.79	530.41	661.65	925.78	1110.7	1276.9
	Ju et al.	342.25	602.12	670.38	997.77	1115.3	1261.1
Interface 2	Present	340.87	592.47	669.05	917.54	1125.0	1308.0
	Ju et al.	342.69	608.78	673.14	1000.8	1161.3	1300.5
Interface 3	Present	340.65	602.98	689.11	958.4	1136.9	1356.9
	Ju et al.	341.10	613.64	700.84	1007.8	1183.7	1426.7
Interface 4	Present	341.05	636.65	698.09	967.19	1141.0	1386.1
	Ju et al.	341.25	647.02	706.29	1010.2	1190.7	1420.2

**Table II: Natural frequencies (Hz) of CCCC plate with a square delamination of side a/2 at different interfaces**

Condition	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
Interface 1	331.51	568.62	648.84	878.54	1088.1	1202.0
Interface 2	332.90	587.60	656.40	943.21	1132.9	1298.6
Interface 3	332.41	595.49	664.31	964.11	1106.7	1346.8
Interface 4	332.67	634.11	700.76	971.20	1119.6	1356.3

So it can be interpreted that the effect of delamination is more dominant in inner surface than other interfaces. The reduction in bending stiffness is more in middle plane. So the delamination damage is more in inner interface.

**Table III: Natural frequencies (Hz) of SSSS plate with a square delamination of side a/3 at different interfaces**

Condition	Location	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
W/o delam		163.58	439.61	525.03	715.44	878.87	1094.3
Interface 1	a/3	161.62	318.60	486.50	614.80	738.40	988.24
	a/2	160.98	345.72	389.20	639.81	690.50	830.50
Interface 2	a/3	162.17	386.70	498.70	656.40	758.40	1006.2
	a/2	161.08	405.52	476.52	686.10	715.00	861.50

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Interface 3	a/3	162.80	426.10	510.80	680.20	791.00	1039.4
	a/2	162.72	413.71	504.17	707.14	755.27	876.40
Interface 4	a/3	163.10	430.70	520.70	706.40	836.40	1060.1
	a/2	163.31	405.63	512.10	710.54	819.40	910.41

The frequency values are small for the SSSS condition compared to the other boundary condition. Also as we move from middle to surface frequency values increases.

**Table IV: Natural frequencies (Hz) of SCCC plate with a square delamination of side a/3 at different interfaces**

Condition	Location	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
W/o delam		327.36	584.6	764.51	942.22	1054.6	1328.7
Interface 1	a/3	324.87	506.71	630.42	888.07	956.80	1230.7
	a/2	324.14	488.70	614.11	879.91	957.56	1176.8
Interface 2	a/3	323.58	536.44	650.80	908.15	1019.7	1286.4
	a/2	326.31	524.82	633.40	918.42	997.81	1198.5
Interface 3	a/3	323.17	556.15	676.14	927.66	1030.8	1297.8
	a/2	326.21	532.71	650.81	941.34	1018.5	1287.1
Interface 4	a/3	324.02	568.32	682.97	934.12	1049.8	1317.5
	a/2	326.76	550.80	676.14	968.04	1033.5	1310.8

The frequency values for SCCC condition are smaller than CCCC condition and higher than other cases. When delamination position move towards the surface frequency value increases.

**Table V: Natural frequencies (Hz) of SSCC plate with a square delamination of side a/3 at different interfaces**

Condition	Location	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
W/o delam		276.05	441.56	640.76	787.76	978.23	1209.74
Interface 1	a/3	272.54	387.90	562.16	730.58	905.86	1146.6
	a/2	271.46	376.10	560.70	724.51	898.70	1128.9
Interface 2	a/3	273.06	401.86	597.81	740.81	917.54	1154.1
	a/2	272.06	396.20	572.51	733.10	913.51	1143.5
Interface 3	a/3	273.96	414.13	614.56	767.32	939.87	1176.5
	a/2	272.81	412.80	615.82	764.50	936.43	1151.5
Interface 4	a/3	274.81	421.82	623.15	782.40	961.56	1198.4
	a/2	273.05	419.60	618.45	776.87	955.87	1174.4

The frequency values of SSCC condition is in between SCCC and SSSC boundary conditions. Also when delamination area increases frequency decreases.

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**Table VI: Natural frequencies (Hz) of SSSC plate with a square delamination of side  $a/3$  at different interfaces**

Condition	Location	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
W/o delam		176.73	382.59	554.25	753.18	947.33	1123.0
Interface 1	a/3	174.16	320.41	418.80	698.40	858.90	1024.5
	a/2	172.81	294.90	476.23	667.86	834.12	950.45
Interface 2	a/3	175.81	338.82	508.10	717.80	886.41	1048.1
	a/2	173.02	312.50	494.86	694.81	866.52	987.56
Interface 3	a/3	176.43	343.80	524.30	734.68	898.99	1059.8
	a/2	175.42	334.31	514.81	718.24	878.81	1038.4
Interface 4	a/3	176.54	376.91	541.8	742.50	921.24	1108.5
	a/2	175.82	352.82	538.23	735.41	916.42	1090.8

The frequency values are in between SSSC and SSSS condition. It is clear that frequency increases as the number of constraints increases.

## VI. CONCLUSION

It is concluded that the natural frequency of delaminated plate is decreases due to decreases in stiffness of delaminated plate. The stiffness degradation of plates is dependent on both delaminated geometry and delamination positions. Natural frequencies of plate will change with boundary condition. It is observed that the natural frequency is greatest in CCCC boundary condition. Frequency decreases in the order SCCC, SSCC, SSSC and lowest in SSSS. That means natural frequency increases with increase in constraints. It is also seen from the above results that natural frequencies are increases as the mode number increases. The mode shape shows that upper segment and lower segment in the delaminated section vibrate in opposite directions. It can also see that as the delamination position away from the surface, frequency increases but the delamination damage is more significant in inner interface compared to surface. Also frequency decreases with the increase in delamination area

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